Problem 1

$$e^{At} \qquad A = \begin{bmatrix} -2 & 3 & -1 \\ 0 & -2 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

$$B = (A - I\lambda) \qquad B = \begin{bmatrix} -2-\lambda & 3 & 21 \\ 0 & -2-\lambda & 4 \\ 2 & 1 & 1-\lambda \end{bmatrix}$$

$$= (-2-\lambda)\begin{vmatrix} -2-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} - 3\begin{vmatrix} 0 & 4 \\ 2 & 1-\lambda \end{vmatrix} + 1\begin{vmatrix} 0 & -2-\lambda \\ 2 & 1 \end{vmatrix}$$

$$= (-2-\lambda)\left((-2-\lambda)(1-\lambda)+4\right) - 3\left(0)\,1-\lambda + 2\times 4\right) - 1\,(0+2(-2-\lambda$$

$$= (-2-\lambda)\left((-2-\lambda)(1-\lambda)+4\right) - \left(-3\times(1-\lambda+8) - 1(2\times -2-\lambda)\right)$$

$$= \left((-2-\lambda)(-2-\lambda)(1-\lambda)+4\right) + 3\times 1-\lambda + 8 - 2\times -2-\lambda$$

$$(-2-\lambda)(-2-\lambda)(1-\lambda) + 4) + 3 - 3\lambda + 8 + 4 + 2\lambda$$

$$(-2-\lambda)(-2-\lambda)(1-\lambda) + 4 - 3\lambda + 15$$

$$(-2-\lambda)(-2-\lambda)(1-\lambda) + 3(\lambda+5)$$

2. $x' = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix} x + e^{-3t}\begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad x(0) = x_0$

$\text{Det} = (-3 \times -3) - (-4 \times 4)$

$= 9 + 16 = 25$

$x^{*} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix}$

$\cancel{a + 0 = -3}$

$\cancel{b + 0 = 4}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -3 & 4 \\ -4 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$-3a - 4b = 1 \qquad 4a - 3b = 0$

$\cancel{4a}$

$-3c - 4d = 0 \qquad 4c - 3d = 1$

$(-3a - 4b = 1)\,4 \qquad b = -\frac{4}{25}$

$(4a - 3b = 0)\,3$

$-12a - 16b = 4 \qquad 4a - 3 \times \frac{-4}{25} = 0$

$12a - 9b = 0 \qquad 25 + 4a + \frac{12}{25} \times 25 = 0 \times 25$

$-25b = 4$

$\frac{-25b}{-25} = \frac{4}{-25} \qquad \frac{100a}{100} = \frac{-12}{100} \qquad a = \frac{-3}{25}$

$b =$

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$$(-3c-4d=0)4$$
$$(4c-3d=1)3$$
$$-12c-16d=0$$
$$12c-9d=3$$

$$12c-9d=3$$
$$-12c-9d=0$$
$$\frac{-18d}{18}=\frac{3}{18}$$

$$d=\frac{-3}{18}$$

$$4c-3\times\frac{-3}{18}=1$$
$$18\times 4c+\frac{9}{18}\overset{\times 18}{=}1\times 18$$
$$72c+9=18$$
$$\frac{72c}{72}=\frac{9}{72} \qquad c=\frac{1}{8}$$

$$\begin{pmatrix}-3 & 4\\ -4 & -3\end{pmatrix}\begin{pmatrix}\frac{-3}{25} & \frac{-4}{25}\\ \frac{1}{8} & \frac{-3}{18}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{matrix}5e^{-3t}\\ -2^{-3t}\end{matrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{matrix}5e^{-3t}\\ -2^{-3t}\end{matrix}$$

3. $x' = y(y-1)(y-2)$

$y' = x(x+1)(x+2)$

$x' = (y^2 - y)(y-2)$

$y^2(y-2) - y(y-2)$

$y^3 - 2y - y^2 - 2y$

$x' = y^3 - 2y^2 - 4y - 2$

$= y^3 - 2y^2 - y - 2$

$y' = (x^2 + x)(x+2)$

$x^2(x+2) + x(x+2)$

$x^3 + 2x^2 + x^2 + 2x$

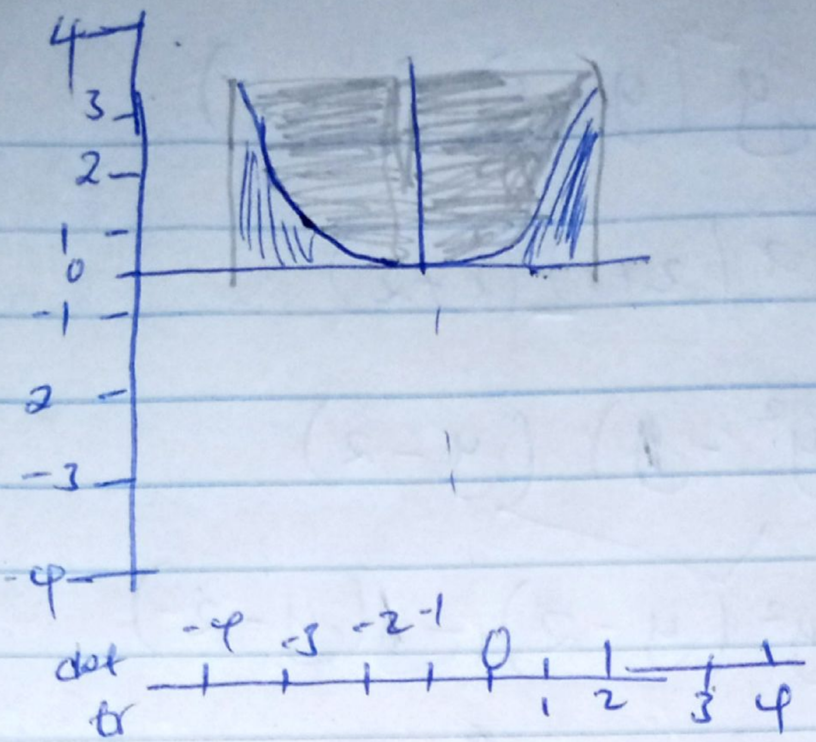
$y' = x^3 + 2x^2 + x + 2$

$\frac{dx}{dy} = y^3 - 2y^2 - y - 2$

$= 3y^2 - 4y - 1$

$\frac{d}{dx} = x^3 + 2x^2 + x + 2$

$= 3x^2 + 4x + 1$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(4) $x' = x^2 - y^2 \quad y' = x - p$

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{bmatrix} f_1 \ (x^2 - y^2) \\ f_2 \ (x - p) \end{bmatrix} = \begin{pmatrix} a^2 y \\ 5x - \sin y \end{pmatrix}$$

$$F_1 \ (x^2 - y^2) = x^2 y$$

$$F_2 \ (x - p) = 5x + \sin y$$

$$J_f \ (x, y) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{pmatrix} 2xy & x^2 \\ 5 & \cos y \end{pmatrix}$$

$$\det J_f \ (x, y) = 2xy \cos y - 5x^2$$